# Using Maximum Likelihood Analysis to Determine the $\pi^+ \rightarrow e^+\nu_e$ Branching Ratio


Anthony Palladino
for the PEN Collaboration

*University of Virginia*
*and*
*Paul Scherrer Institute*


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# The PEN Experiment

- Precision Measurement of the $\pi^+ \rightarrow e^+\nu$ branching ratio.

$$B = \frac{\Gamma(\pi^+ \rightarrow e^+\nu_e(\gamma))}{\Gamma(\pi^+ \rightarrow \mu^+\nu_\mu(\gamma))} = \left(\frac{g_e}{g_\mu}\right)^2 \left(\frac{m_e}{m_\mu}\right)^2 \frac{\left(1-m_e^2/m_\mu^2\right)^2}{\left(1-m_\mu^2/m_\pi^2\right)^2} (1 + \delta R)$$

$B_{calc} = (1.2352 \pm 0.0005) \times 10^{-4}$ Marciano & Sirlin, [PRL **71**, 3629 (1993)]

$B_{calc} = (1.2354 \pm 0.0002) \times 10^{-4}$ Finkemeier, [Phys. Lett. B **387**, 391 (1996)]

$B_{calc} = (1.2352 \pm 0.0001) \times 10^{-4}$ Cirigliano & Rosel, [PRL **99**, 231801 (2007)]

$B_{exp} = (1.230 \pm 0.004) \times 10^{-4}$ Experiment World Average (Current PDG)

Lepton Universality: W. Loinaz, et. al., Phys. Rev. D **65**, 113004 (2004) [hep-ph/0403306]

$$\left(\frac{g_e}{g_\mu}\right)_\pi = 1.0021 \pm 0.0016$$

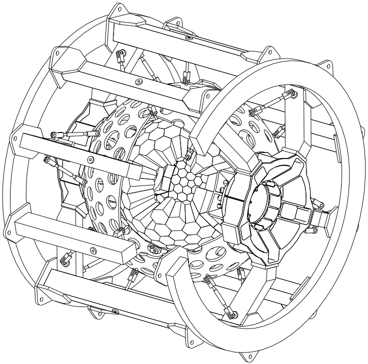
| Our Goal: | $\frac{\Delta B_{exp}}{B_{exp}} \leq 5 \times 10^{-4}$ |
|---|---|

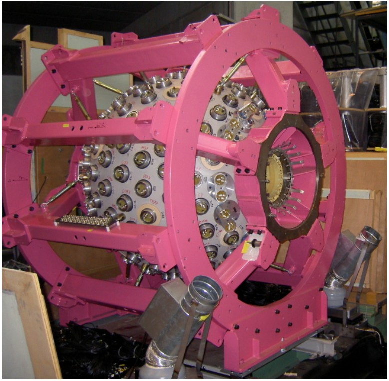
# Mass Limits on Leptoquark and Supersymmetric Particles

We will be able to give lower bounds on the masses of some hypothetical particles in theories beyond the standard model.
Following the calculations in Shanker, NP B204 (82) 375:

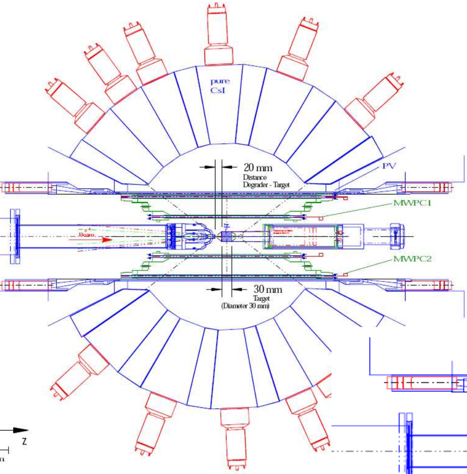
| Particle | Projected Lower Bound | Current Bounds |
|---|---|---|
| Charged Higgs Boson: | $m_H > 6.9$ TeV | $m_H > 2$ TeV |
| Pseudoscalar Leptoquark: | $m_p > 3.8$ TeV | $m_p > 1.3$ TeV |
| Vector Leptoquark: | $M_G > 630$ TeV | $M_G > 220$ TeV |

# PEN Detector





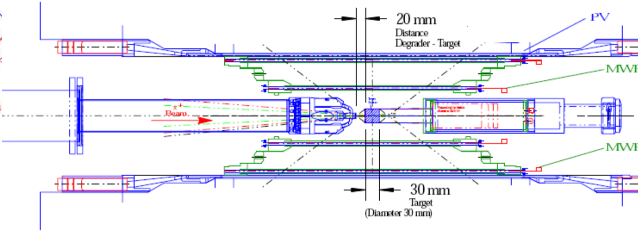
# Experimental Method




Detector cross-sections

2008 Run

Wedged Degrader



# Maximum Likelihood Analysis

## Model: Probability Density Functions

One combined p.d.f. encompassing many observables and processes.

$$\mathcal{L}(\mathbf{x};\mathbf{N}) = \prod_{i=1}^{n} F_i(x_i;\mathbf{N}) \quad \text{where} \quad \mathbf{N} = N_{j=1,\ldots,m}$$

$$F_i(x_i;\mathbf{N}) = \sum_{j=1}^{m} f_j(x_i;N_j)$$

**$n$ Observables ($x_i$)**

- Time between $\pi^+$ and $e^+$
- Total Positron Energy
- "Probability" of Pile-up

$$\text{“}P\text{“}_{\text{pile-up}} = \ln\left[\sum_{k=1}^{\ell} e^{-|dt_k|/\tau_\mu}\right]$$

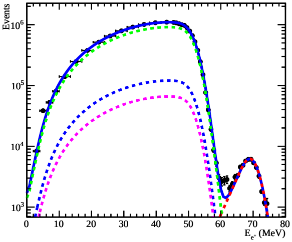
- Pion Decay Vertex

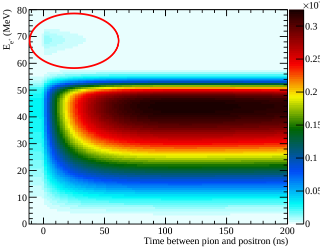
**$m$ Processes with normalization ($N_j$)**

- $N_{\text{p2e}}$, $\pi^+ \to e^+$ (p2e)
- $N_{\text{mich}}$, $\pi^+ \to \mu^+ \to e^+$ (Michel)
- $N_{\text{acc}}$, Accidentals / Pile-up
- $N_{\text{dif}}$, Pion Decays-in-flight

# Model: Probability Density Functions

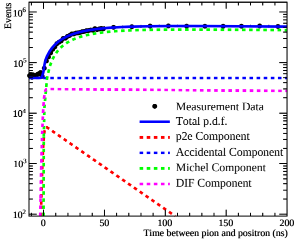
$$\mathcal{L}\left(\mathbf{E},\mathbf{t};N_{p2e},N_{mich},N_{acc},N_{dif}\right)=f_1\left(\mathbf{E};\mathbf{N}\right)f_2\left(\mathbf{t};\mathbf{N}\right)$$



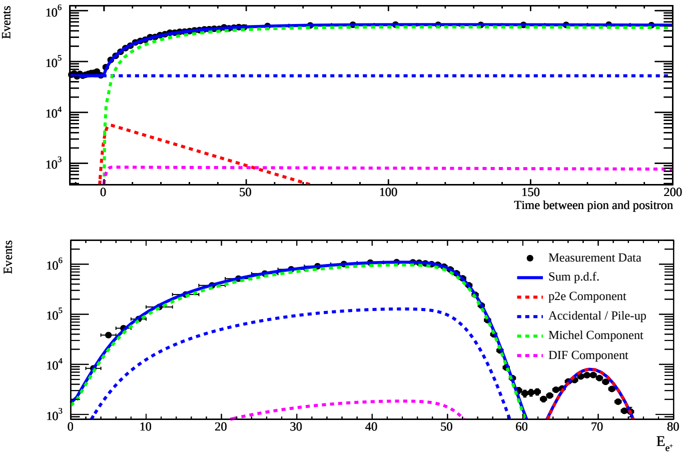




Maximization Techniques:

- Binned vs. Unbinned
- Maximum Likelihood Fit
- $\chi^2$ Fit
- Negative Log Likelihood, $\ell = -\ln\mathcal{L}$



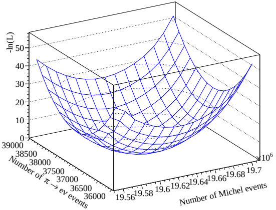
# Measurement: Data

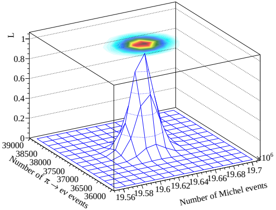


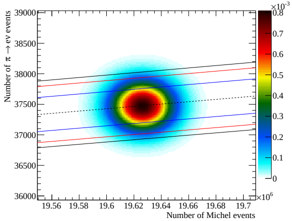

Variable Binning → Improvement in calculation speed.

# Techniques: Negative Log Likelihood

$\ell = -\ln\mathcal{L}$ $\mathcal{L} = e^{-\ell}$ $N_{\mathrm{p2e}}$ vs. $N_{\mathrm{michel}}$




Use of standard software libraries MINUIT, MIGRAD, HESSE, and MINOS on $\ell$. Use of ROOFIT and ROOSTATS to set up model p.d.f.

# Preliminary Results

Symmetric Confidence Intervals:

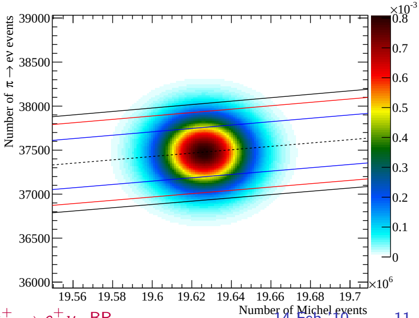
(This is only 8.1% of 2008 data $\rightarrow \frac{1}{\sqrt{N_{\mathrm{p2e}}}} \simeq 6 \times 10^{-3}$ )

68.3% ($1\sigma$): $B = (1.228 \pm 0.142) \times 10^{-4}$

$$\frac{\Delta B}{B} = 0.116$$

Dominated by systematic uncertainties... no well defined DIF p.d.f. yet.

Proper confidence levels are calculated using the Likelihood Ratio, $R = \frac{\mathcal{L}(\mathbf{x};\mathbf{N})}{\mathcal{L}(\mathbf{x};\hat{\mathbf{N}})}$ where $\hat{N}$ maximizes $\mathcal{L}(\mathbf{x};\mathbf{N})$. Calculate $R$ for all $\mathbf{N}$ and rank the $N_j$ values according to their $R$ values. $N_j$ are added to the confidence region first until $\int \mathcal{L}(\mathbf{x};\mathbf{N})\, d\mathbf{x}$ over the confidence region reaches our desired confidence level.



# Conclusions

Maximum Likelihood benefits:

- Provides a unique, unbiased, minimum variance estimate (for a large enough sample).
- Practical, tractable approach via product p.d.f.'s
- Use all the information in the data to determine the parameters; minimal cuts.

Drawbacks:

- Critical dependence on p.d.f.

# PEN Experiment collaboration members:

L.P. Alonzi,[a] V. A. Baranov,[c] W. Bertl,[b] M. Bychkov,[a] Yu.M. Bystritsky,[c] E. Frlež,[a] V. Kalinnikov,[c] N.V. Khomutov,[c] A.S. Korenchenko,[c] S.M. Korenchenko,[c] M. Korolija,[f] T. Kozlowski,[d] N.P. Kravchuk,[c] N.A. Kuchinsky,[c] M.C. Lehman,[a] D. Mekterović,[f] D. Mzhavia,[c,e] A. Palladino,[a,b] D. Počanić,[a⋆] P. Robmann,[g] O.A. Rondon-Aramayo,[a] A.M. Rozhdestvensky,[c] T. Sakhelashvili,[b] V.V. Sidorkin,[c] U. Straumann,[g] I. Supek,[f] Z. Tsamalaidze,[e] A. van der Schaaf,[g⋆] E.P. Velicheva,[c] V.V. Volnykh,[c]

[a] *Dept of Physics, Univ of Virginia, Charlottesville, VA 22904-4714, USA*
[b] *Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*
[c] *Joint Institute for Nuclear Research, RU-141980 Dubna, Russia*
[d] *Institute for Nuclear Studies, PL-05-400 Swierk, Poland*
[e] *IHEP, Tbilisi, State University, GUS-380086 Tbilisi, Georgia*
[f] *Rudjer Bošković Institute, HR-10000 Zagreb, Croatia*
[g] *Physik Institut der Universität Zürich, CH-8057 Zürich, Switzerland*

blue = Graduate Student
⋆ = Spokesperson